Ch. 1 – Introduction:

Units and Measurement

Scientific Notation

- In physics we encounter very large and very small numbers.
- For example:
  - Speed of light: c = 300,000,000 m/s
  - Planck's constant:

    h = .000000000000000000000000000000663 J·s
- We want a better way to write these numbers.

Scientific Notation

- Numbers in scientific notation are written in this form:

$$a.bc \times 10^{d}$$

- d>0 → decimal point moves to the right to get back to ordinary notation (large numbers)
- d<0 → decimal point moves to the left to get back to ordinary notation (small numbers)
- Examples:
  - Speed of light: $c = 3.00 \times 10^{8}\ \frac{m}{s}$
  - Planck's constant: $h = 6.63 \times 10^{-34} J \cdot \text{s}$
  - Wavelength of green light: $\lambda = 5 \times 10^{-9} m = .000000005\ \text{m}$

Multiplying and Dividing

- Scientific notation also makes multiplying and dividing large and small numbers easier.
- For example:

- $(3.2 \times 10^3)(2.1 \times 10^{-2}) = (3.2 \times 2.1) \times 10^{3+(-2)}$
  $= 6.72 \times 10^1 = \boxed{67.2}$

- $\frac{3.2 \times 10^3}{2.1 \times 10^{-2}} = \left(\frac{3.2}{2.1}\right) \times 10^{3-(-2)} = 1.524 \times 10^5$
  $= \boxed{152400}$

Significant digits

- Not all digits returned by your calculator necessarily mean anything
- Measured values are limited in resolution: multiplying or dividing them cannot make them more precise (i.e. accurate to more digits)
- If you had three nonzero digits in the numbers you plugged in, then only three digits of the output are meaningful.
- In general:
  - (number of significant digits out) = (**smallest** number of significant digits among the input values)
- For example:
  - Multiply **(3.23)x(2.4789) = 8.006847**
  - Only the first three digits are significant: **8.01** (3 digits, rounding last digit to nearest value)

Units

- We will primarily use the **metric system**
  - (also known as the **SI** (standard international) or **MKS** (meter-kilogram-second) system)
- Basic units for now:
  - **Meters (m)** for distance or length
  - **Kilograms (kg)** for mass
  - **Seconds (s)** for time
- Later we will add two more basic units:
  - **Kelvin (K)** for temperature
  - **Coulomb (C)** for charge
- All other units we need can be built out of combinations of these

Adjusting unit sizes

- The sizes of our units can be adjusted by adding prefixes.
- The prefixes we will see over the course of the year are:
  - Kilo (k) = 10^{3} (one thousand)
  - Centi (c) = 10^{-2} (one hundredth)
  - Milli (m) = 10^{-3} (one thousandth)
  - Micro (μ) = 10^{-6} (one millionth)
  - Nano (n) = 10^{-9} (one billionth)
  - Pico (p) = 10^{-12} (one trillionth)
- For example:

  10 kg = 10,000 g = 10^{7} μ g

  1 cm = 10 mm = .01 m = 10^{-5} km

Other Units

- Other (non-MKS) units will occasionally be used, such as:
  - Feet (ft) and miles for distance
  - Pounds (lbs) for weight or force
  - Celsius (°C) for temperature
  - Liters (L) for volume
  - Pascals or atmospheres for pressure
- In general, we will convert these into MKS units before using them.
- All quantities being plugged into an equation **must have consistent units.**
- For example, you can't combine a distance in miles with a distance in meters in the same equation.

Unit Conversion

- **General strategy:**
  - Look up conversion factor(s) in book or in formula sheet
  - Form the conversion into a ratio and multiply the original quantity by this ratio, so that the unwanted units cancel. (see examples below).
  - Multiple conversion factors may be needed
  - Powers of conversion factors may be needed.

Example: Unit conversion

- 2.5 ft equals how many meters?
  - Conversion factor: 1 m = 3.28 ft
  - Form the ratio $\left(\frac{1m}{3.28\ ft}\right)$. Since the top and bottom are equal, this ratio really just equals 1.
  - Multiply this ratio times the original value and cancel the unwanted units:
  - $(2.5\ ft) \cdot \left(\frac{1m}{3.28\ ft}\right) = \left(\frac{2.5\ \cancel{ft}}{3.28\ \cancel{ft}}\right) \cdot 1\text{m} = \boxed{.762\ \text{m}}$

  - **Note** that units can be multiplied, divided, and canceled, just like numbers.

Are Units important?

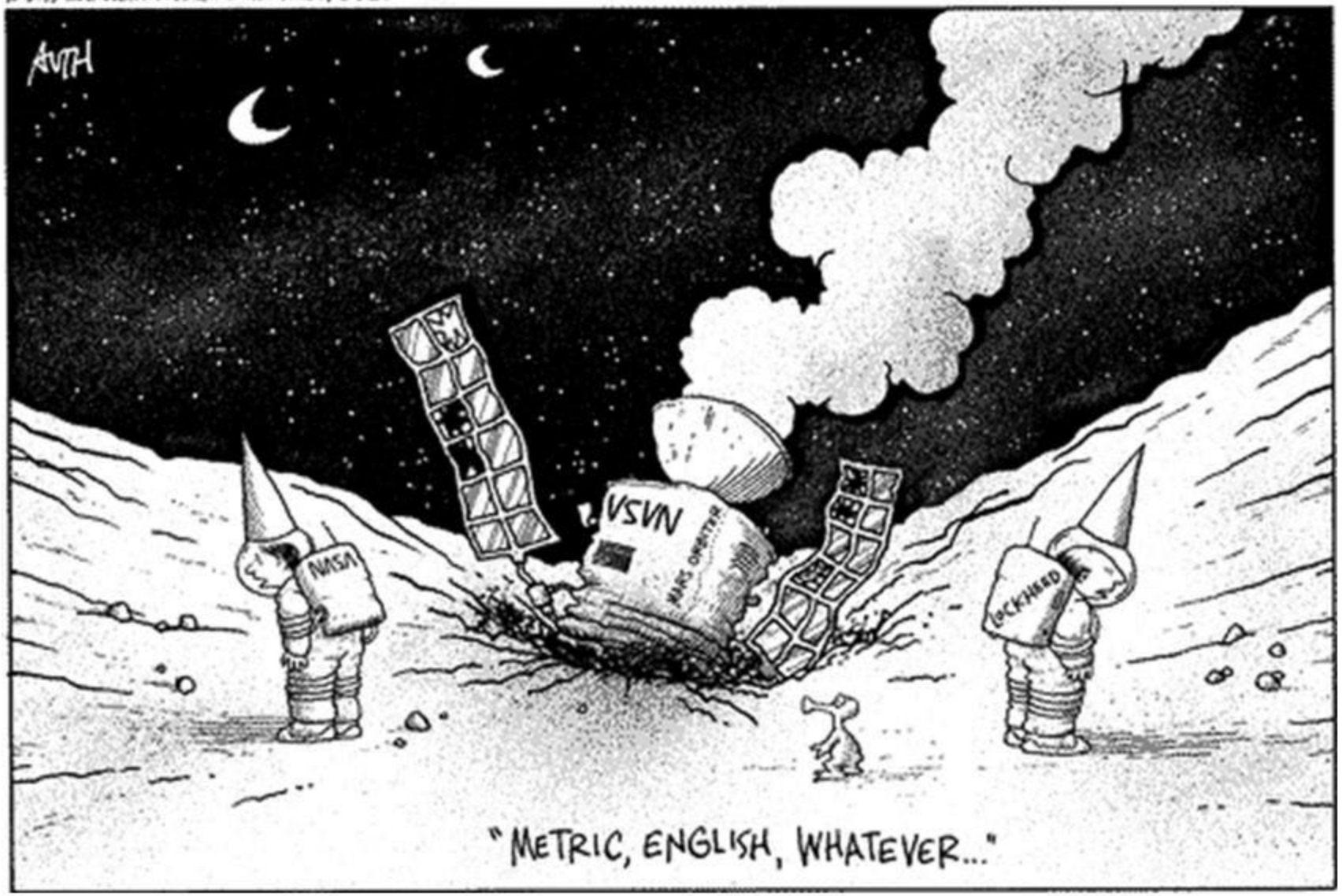
On 9/23/99, $125,000,000 Mars Climate Orbiter entered Mar's atmosphere 100 km (62 miles) lower than planned and was destroyed by heat.

The 'root cause' of the loss of the spacecraft was the failed translation of English units into metric units in a segment of ground-based, navigation-related mission software

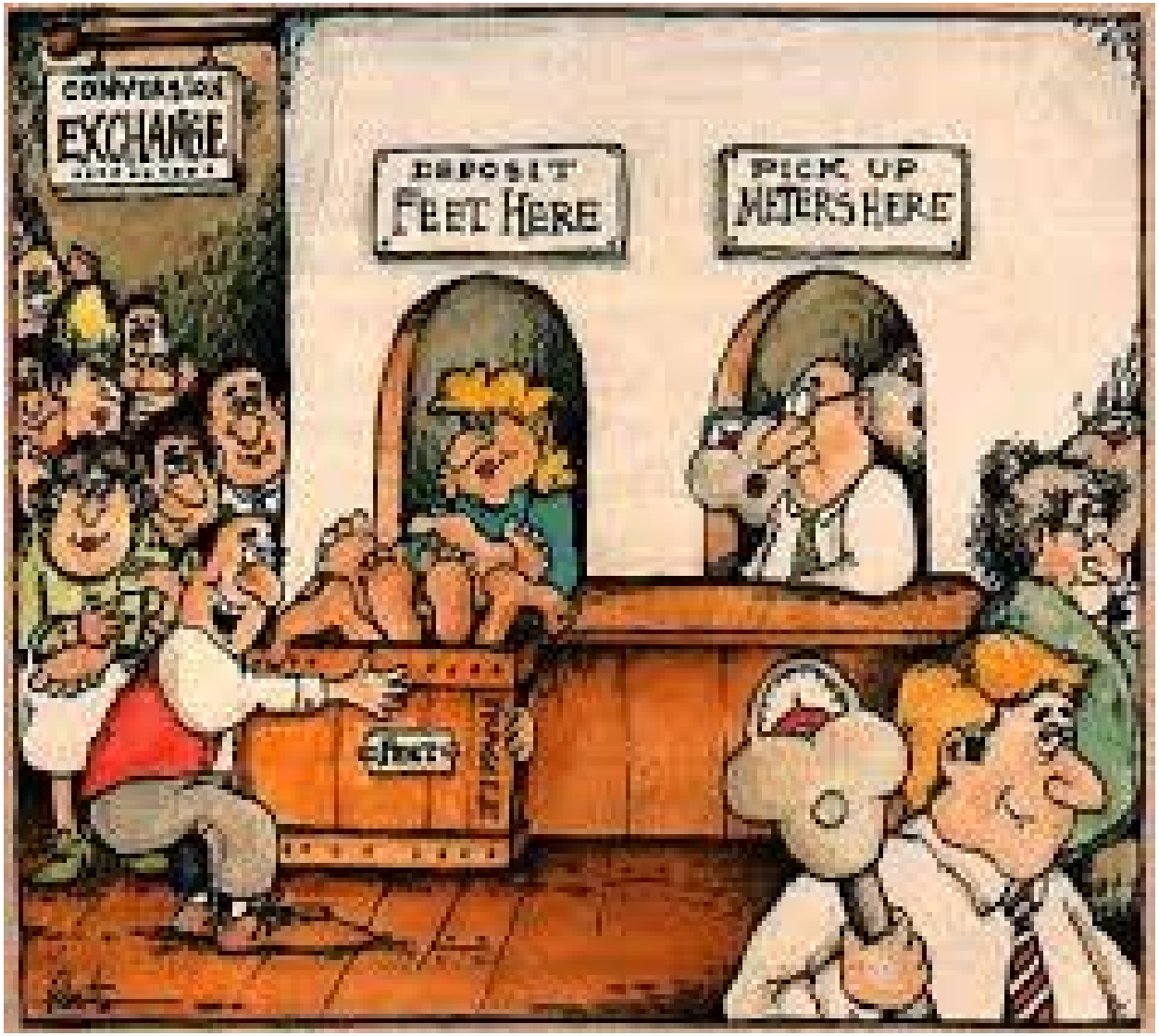
1 lb ≠ 1 N

1 lb = 4.45 N

The importance of unit conversion



Remember the Mars Climate Orbiter incident from 1999?



Fortunately, conversion is easy and convenient

Example: Unit conversion

- Convert 2500 m into miles.

  - Conversion factors: 1 km = .621 mile, 1 km =10^3 m

  - $(2500\ m) \cdot \left(\frac{1\ km}{1000 m}\right) \cdot \left(\frac{.621\ miles}{1\ km}\right) = \left(\frac{(2500)(.621)}{10^3}\right) mile$

    $= \boxed{1.55 \text{ miles}}$

Example: Unit conversion

- What is 2000 ft/min when converted to m/s?
  - Conversion factors: 1 m = 3.28 ft , 1 min = 60 s
  - $(2000\ \frac{\cancel{ft}}{\cancel{min}}) \cdot \left(\frac{1\ m}{3.28\ \cancel{ft}}\right) \cdot \left(\frac{1\ \cancel{min}}{60\ s}\right) = \left(\frac{2000}{(3.28)(60)}\right)\frac{m}{s}$

$= \boxed{10.163\ \frac{m}{s}}$

Example: Unit conversion

- Convert the **area** 3 ft^2 into m^2.
  - Conversion factor: 1 m = 3.28 ft
  - $(3\ ft^2) \cdot \left(\frac{1\ m}{3.28\ ft}\right)^2 = \boxed{.279\ m^2}$
- Convert the **volume** 3 ft^3 into m^3.
  - $(3\ ft^3) \cdot \left(\frac{1\ m}{3.28\ ft}\right)^3 = \boxed{0.0850\ m^3}$

Supplementary material for use in lab

Describing Data: Mean and Standard Deviation

- Suppose we are interested in some physical variable, x.
- Make n measurements. This results in n data points:
  - $x_1, x_2, x_3, \ldots\ldots, x_n$
- We want a convenient way of summarizing this data.
- Most common way: give a measure of the **center** of the data (the **mean**) and a measure of the **spread** of the data (the **standard deviation**).
- First, some notation.

Notation: Summation symbol

- Denote a generic data point by x_i, where $i = 1, 2, 3, \ldots, n$
- Define the summation symbol:

  $\Sigma =$ capitol Greek letter Sigma

  ⟶ take sum of all values of whatever is to the right

- **Example:** $x_1 = 3, \quad x_2 = 4, \quad x_3 = 2$
  - $\Sigma\, x = x_1 + x_2 + x_3 = 3 + 4 + 2 = 9$
  - $\Sigma\, x^2 = x_1^2 + x_2^2 + x_3^2 = 3^2 + 4^2 + 2^2 = 9 + 16 + 4 = 29$
  - $(\Sigma\, x)^2 = (x_1 + x_2 + x_3)^2 = (3 + 4 + 2)^2 = 9^2 = 81$

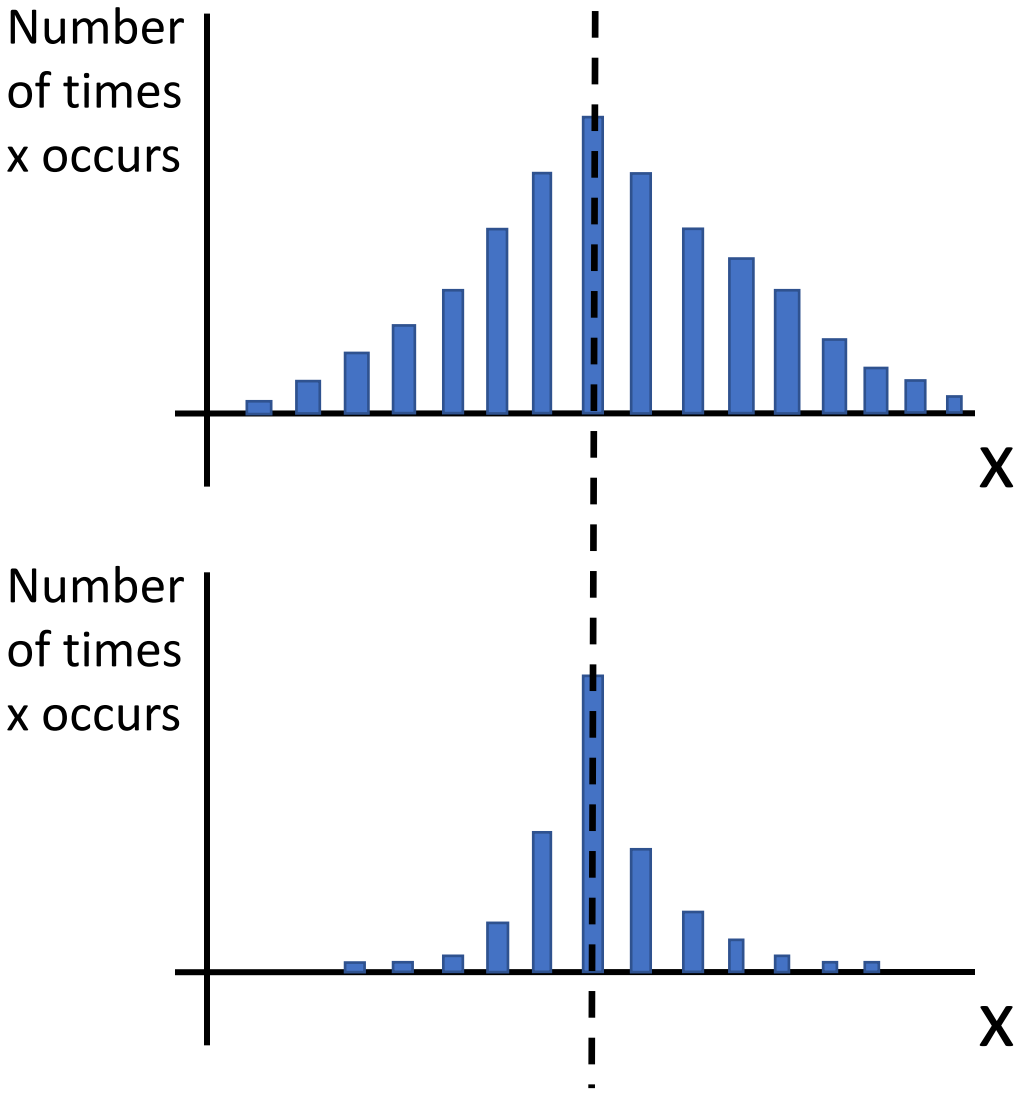
Mean

- The mean or average measures the center of a distribution of data, or equivalently it gives the "most typical" value.
- **Definition:**

$$\bar{x} = \frac{1}{n}\sum x = \frac{1}{n}(x_1 + x_2 + x_3 + \ \ldots\ldots + x_n)$$

- **Example:** $x_1 = 3, \quad x_2 = 4, \quad x_3 = 2$
  - $\bar{x} = \frac{1}{3}(3 + 4 + 2) = \frac{9}{3} = \boxed{3}$

Standard Deviation



- These two data sets have the same mean, but are clearly different.
- In the bottom set, most data points are clustered close to the mean.
- In the top one, many points are far from the mean.
- Clearly the mean tells us more about a typical data point in the bottom set than in the top.
- We need a way to distinguish between these. This will be the **standard deviation**.

Standard Deviation

- The **standard deviation** measures the spread of the data away from the mean, or the width of the distribution.
- Can think of the standard deviation as the average distance of a typical data point from the mean. (This needs to be taken with a grain of salt: see next slide)

- Notation: standard deviation is s (or sometimes σ_{n-1})

- Note we are using the **sample** standard deviation; the **population** standard deviation is slightly different.

Standard Deviation

- Suppose we take literally the statement "standard deviation as the average distance of a typical data point from the mean". What happens?
- Distance of data point x from the mean: $x - \overline{x}$
- Average distance:

$$\frac{1}{n}\sum(x - \overline{x}) = \frac{1}{n}\sum x - \frac{1}{n}\sum \overline{x} = \frac{1}{n}\sum x - \frac{1}{n}(n\overline{x})$$
$$= \overline{x} - \overline{x} = 0$$

- So we make two changes:
  - We square before averaging, to prevent positive and negative values from canceling.
  - For technical reasons we divide by n-1, rather than n. (One value is not independent once the mean is fixed.)

Standard Deviation

- Definition of (sample) standard deviation:

$$s = \sqrt{\frac{1}{n-1}\sum (x - \overline{x})^2}$$

- The square root is needed to get back to the original units.
- An alternate (equivalent) expression is often useful:

$$s = \sqrt{\overline{x^2} - \overline{x}^2}$$

Order of operations is important: In first term, square then average. In second term, average, then square.

Standard error

- In labs you may be asked to find the **standard error** or **margin of error:**

$$\delta x = \frac{s}{\sqrt{n}}$$

- δx gives a measure of how well the mean is to be trusted: if it is small, the **estimated mean** (the sample mean) from the data is likely to be close to the **true mean** (the population mean).

- The outcome of an experiment is often reported as a **confidence interval**: $\overline{x} \pm \delta x$

- This indicates that the true mean is likely to be within distance δx of the estimated sample mean $\overline{x}$

Example

- A voltage is measured n = 5 times. The measured values are: $\mathrm{V} = 10V, 12V, 11V, 9V, 11V$

- Form a table: Fill data in, then take sum of column in last row.

V	$V - \overline{V}$	$(V - V)^2$
10	-.6	.36
12	1.4	1.96
11	.4	.16
9	-1.6	2.56
11	.4	.16
53	0	5.2

ΣV $\Sigma (V - \overline{V})$ $\Sigma (V - \overline{V})^2$

$$\overline{V} = \frac{1}{n}\Sigma V = \frac{1}{5}(53) = \boxed{10.6\ V}$$

$$s = \sqrt{\frac{1}{n-1}\sum (V - \overline{V})^2}$$

$$= \sqrt{\frac{5.2}{4}} = \boxed{1.14\ V}$$

$$\delta V = \frac{s}{\sqrt{n}} = \frac{1.14}{\sqrt{5}} = \boxed{.51\ V}$$

Measurement uncertainty

- In addition to the **statistical uncertainty** (standard error) due to multiple measurements, there is also **measurement uncertainty** due to limited resolution of the measuring device.
- For example if measuring a length with a ruler marked in multiples of ¼ inch, the minimum resolution is 1/8 inch.
- In general, minimum measurement error is ½ spacing between markings on measurement device.
- Any decimal places beyond this resolution limit are meaningless.
- Multiple sources of error are added as follows:

$$total\ error = \sqrt{(statistical\ error)^2 + (\ measurement\ error)^2 + \ \ldots\ldots}$$

Measurement uncertainty

- Often error is expressed as **percent error** or **percent difference**.
- The **<u>percent error</u>** from an accepted value is:

$$\%\ error = \frac{|accepted\ value\ - measured\ value|}{accepted\ value} \times 100\%$$

- **Example**: An experiment measures a value of $1.7\text{x}10^{-19}$ C for the charge of an electron. What is its percent error from the accepted value of $1.6\text{x}10^{-19}$ C ?

$$\%\ error = \frac{|1.7\text{x}10{-}19 - 1.6\text{x}10{-}19|}{1.6\text{x}10{-}19} \times 100\% = \boxed{6.25\%}$$

Measurement uncertainty

- The **percent difference** between two measured values is:

$$\%\ diff. = \frac{|value\ 1 - value\ 2|}{\left(\frac{value\ 1 + value\ 2}{2}\right)} \times 100\%$$

- **Example**: Two experiments measure values of 1.7×10^{-19} C and 1.8×10^{-19} C for the charge of an electron. What is its percent difference between these values?

$$\%\ error = \frac{|1.7\text{x}10{-}19 - 1.8\text{x}10{-}19|}{\left(\frac{1.7\text{x}10{-}19 + 1.8\text{x}10{-}19}{2}\right)} \times 100\%$$

$$= \frac{|1.7\text{x}10{-}19 - 1.8\text{x}10{-}19|}{1.75\text{x}10{-}19} \times 100\% \quad = \boxed{5.71\%}$$